

Warm inflation in loop quantum cosmology: a model with a general dissipative coefficient

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A general form of warm inflation with the dissipative coefficient $\Gamma = \Gamma_0 (\phi/\phi_0)^n (T/\tau_0)^m$ in loop quantum cosmology is studied. In this case, we obtain conditions for the existence of a warm inflationary attractor in the context of loop quantum cosmology by using the method of stability analysis. The two cases when the dissipative coefficient is independent ($m = 0$) and dependent ($m \neq 0$) on temperature are analyzed specifically. In the latter case, we use the new power spectrum which should be used when considering temperature dependence in the dissipative coefficient. We find that the thermal effect is enhanced in the case $m > 0$. As in the standard inflation in loop quantum cosmology, we also reach the conclusion that quantum effect leaves a tiny imprint on the CMB sky.

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I. INTRODUCTION

Inflation, as a necessary supplement to the standard cosmological model, can successfully explain many long-standing problems such as horizon and flatness. Inflation is also a good mechanism to explain the large scale features of the universe [1–3]. The standard inflation needs a reheating period to end the inflation and start the radiation dominated phase. Another type of inflation called “warm inflation” was proposed by Berera and Fang [4]. From then many works have been done in the context of warm inflation, such as tachyon warm inflationary model [5] and natural warm inflation [6], but most of them are focused on a classical universe case. In the scenario of warm inflation, the inflation of the universe is accompanied by continuous radiation producing, so the universe is hot during inflation and can go into radiation dominated phase smoothly. In order to realize the acceleration of the universe, the dominant energy is still the potential of the inflaton. The radiation producing is due to the interaction between the inflaton and other fields. After considering the dissipative interaction, a dissipative term is contained in the equation of motion of the inflaton. Different dissipative coefficient (denoted as Γ) can be obtained when considering different microscopic models. We will use a most general form of dissipative coefficient $\Gamma = \Gamma_0 (\phi/\phi_0)^n (T/\tau_0)^m$ by concluding all the cases and try to give a general picture. Besides these, warm inflation can also eliminate the “ η -problem” [7] and decrease the overlarge amplitude of inflaton in standard inflation [8]. A most discriminative feature of the warm inflation is that the main contribution to the density perturbation is the thermal fluctuations rather than vacuum fluctuation [4, 9, 10]. The power spectrum in warm inflation is analyzed mostly in the case that the dissipative

coefficient is independent on temperature [9, 10], however, considering the microphysical basis of warm inflation in the very early universe, the dissipative coefficient is dependent on temperature in most cases and deserve more research. The new scalar perturbation spectra for $\Gamma = \Gamma(\phi, T)$ case is given by Chris Graham and Ian G. Moss in [11]. We use the new scalar perturbation spectra when analyzing the temperature dependent case.

On the other hand, loop quantum gravity (LQG) is a mathematically well-defined, nonperturbative, and background independent quantization of gravity [12]. The space-time geometry in LQG is discrete when approaching the Planck scale and become continuous in a large eigenvalue limit. LQG is a good and pioneering scheme to unify quantum mechanics and gravity. Loop quantum cosmology (LQC) is the application of LQG in the homogeneous and isotropic universe [13–15]. LQC inherits quantization scheme and techniques from LQG but focuses on symmetry reduced models and solves many longstanding problems. The singularity in general relativity is replaced by a cosmological bounce in LQC. The underlying dynamics of LQC is the discrete quantum difference equation of quantum geometry which is not easy to solve. Fortunately, there are two approaches to solve this problem, one is using sophisticated numerical methods [16], and the other one is using semiclassical states to construct an effective theory of LQC by incorporating quantum corrections to the classical dynamics. Generally speaking, there are two kinds of quantum corrections, namely inverse-volume correction and holonomy correction [17]. In both cases, one can get a modified Friedmann equation by making Hamilton analysis. The modified Friedmann equation is very different from the classical one but can reduce to classical dynamics when the quantum effect is quite weak. In this paper, we use holonomy correction Friedmann equation to investigate warm inflation in loop quantum cosmology (Warm-LQC). With holonomy correction, the Friedmann equation acquires a quadratic density modification when the energy scale is comparable to the critical en-

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ergy density ρ_c ($\rho_c \simeq 0.41\rho_{pl}$, $\rho_{pl} = m_p^4$, $m_p^2 = G^{-2}$) [18]. That the universe can undergo a big bounce when the energy density equals the critical density can be seen from the modified Friedmann equation. Cosmological inflation naturally happens after the big bounce, and a new phase named super-inflation [19] is found before the normal inflation, which is a character of LQC. The standard inflationary model in LQC was fully researched in Ref. [20, 21]. The warm inflationary model in LQC was also considered in Refs. [22, 23], which are a good try to combine warm inflation with LQC. But for simplicity, they only discuss the case when the dissipative coefficient is a constant. Thermal fluctuations were treated earlier in loop cosmology in [24] in a noninflationary setting and they restricted themselves on the closed universe models. They assumed a purely thermal origin for the fluctuations and obtained a power spectrum through the partition function. However they can't get a nearly scale-invariant spectrum in classical universe. In order to get a nearly scale-invariant spectrum, they make use of inverse-volume correction in LQC and the problem in the classical universe is eliminated.

In this paper we try to give a general picture of a warm inflationary model in LQC. We use the method of stability analysis to obtain the slow-roll conditions in Warm-LQC when both the potential and the dissipative coefficient might be dependent on temperature. We study two cases when the dissipative coefficient is independent and dependent on temperature and give a workable example in both cases. What we should note here is that in both cases we treat the perturbations in Warm-LQC following the way in [20, 22], for the quantum effect is not obvious when horizon crossing as we will argue later in the paper. More details and consistent works about perturbations in LQC is beyond the scope of our works and can be found in [25, 26]. In the temperature dependent case, we take the new form of the power spectrum which should be used when the dissipative coefficient is temperature dependent. We use the Wilkinson Microwave Anisotropy Probe (WMAP) data to constrain parameters and detect the order of magnitudes of quantum effect.

The outline of the paper is as follows. In the next section we introduce the effective theory of LQC. In Sec. III the dynamics of Warm-LQC is briefly introduced. We analyze the slow-roll conditions in Warm-LQC in Sec. IV. The two cases when the dissipative coefficient is independent and dependent on temperature are calculated respectively in Sec. V and Sec. VI. Finally, we draw the conclusions in Sec. VII.

II. LOOP QUANTUM COSMOLOGY

In this section, we will introduce the effective theory of LQC based on holonomy correction in the flat model of universe. In LQG, the phase space of classical general relativity is expressed in terms of $SU(2)$ connection A_a^i and density-weighted triads E_i^a . After the symmetry reduc-

tion and the gauge fixing from LQG, the only remaining degrees of freedom in the phase space of LQC are the conjugate variables of connection c and triad p , which satisfy Poisson bracket $\{c, p\} = \frac{1}{3}\gamma\kappa$, where $\kappa = 8\pi G$, γ is the Barbero-Immirzi parameter (γ is set to be $\gamma \simeq 0.2375$ by the black hole thermodynamics) [27]. In the Friedmann-Robertson-Walker (FRW) cosmology, the two conjugate variables can be expressed as

$$c = \gamma\dot{a}, \quad p = a^2, \quad (1)$$

where a is the FRW scale factor. The classical Hamiltonian constraint in terms of the connection and triad is given by [15]

$$\mathcal{H}_{cl} = -\frac{3}{\kappa\gamma^2}\sqrt{p}c^2 + \mathcal{H}_M, \quad (2)$$

where \mathcal{H}_M is the matter Hamiltonian. Using holonomy correction to modify the gravity part in the classical Hamiltonian constraint, the effective Hamiltonian constraint is given by [17]

$$\mathcal{H}_{eff} = -\frac{3}{\kappa\gamma^2\bar{\mu}^2}\sqrt{p}\sin^2(\bar{\mu}c) + \mathcal{H}_M. \quad (3)$$

where $\bar{\mu}$ corresponds to the dimensionless length of the edge of the elementary loop over which the holonomies are computed, and the area is $\mathcal{A} = \bar{\mu}^2 p = \alpha l_{pl}^2$, where α is of order of unity and $l_{pl} = \sqrt{\hbar G}$ is the Planck length. Using the Hamiltonian constraint (3) one can get the Hamiltonian equation for p :

$$\dot{p} = \{p, \mathcal{H}_{eff}\} = -\frac{\kappa\gamma}{3}\frac{\partial\mathcal{H}_{eff}}{\partial c} = \frac{2a}{\gamma\bar{\mu}}\sin(\bar{\mu}c)\cos(\bar{\mu}c), \quad (4)$$

which combined with Eq. (1) implies that

$$\dot{a} = \frac{1}{\gamma\bar{\mu}}\sin(\bar{\mu}c)\cos(\bar{\mu}c). \quad (5)$$

Furthermore, the vanishing Hamiltonian constraint $\mathcal{H}_{eff} \approx 0$ yields

$$\sin^2(\bar{\mu}c) = \frac{\kappa\gamma^2\bar{\mu}^2}{3a}\mathcal{H}_M. \quad (6)$$

From Eqs. (5) and (6), the effective Friedmann equation results in

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \quad (7)$$

where $H = \dot{a}/a$ is the Hubble rate and the critical density ρ_c is given by

$$\rho_c = \frac{3}{\kappa\gamma^2\bar{\mu}^2 a^2} = \frac{\sqrt{3}}{32\pi^2\gamma^3}\rho_{pl} \simeq 0.41\rho_{pl}, \quad (8)$$

where $\rho_{pl} = G^{-2}$ is the Planck density and $\alpha = 4\sqrt{3}\pi\gamma$ [18] is used. Compared with the classical Friedmann equation, a ρ^2 correction term is added to the effective Friedmann in LQC, which implies the Hubble rate vanishes when $\rho = \rho_c$ and the universe undergoes a turnaround in the scale factor instead of singularity. When $\rho \ll \rho_c$, the modification term is negligible and the classical one is recovered.

III. BASIC EQUATIONS OF WARM-LQC

We consider a spatially flat, homogeneous universe dominated by a scalar field ϕ (inflaton) and radiation produced by the interaction of inflaton with other fields which are subordinate. The interaction existing during inflation seems more natural than the assumption in standard inflation that the inflaton is an isolated, noninteracting field, so instead of a steep supercooling phase in standard inflation, the universe has a temperature T during warm inflation. The scalar inflaton field must be potential energy dominated to realize inflation. In the Warm-LQC scenario, the evolution of the inflaton is governed by both the potential $V(\phi, T)$ and the dissipative coefficient $\Gamma(\phi, T)$

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_\phi = 0, \quad (9)$$

where the subscripts denote a derivative. For simplicity, Γ is often set to be a constant in some papers [22, 23, 28]. Considering some concrete model of the interaction between inflaton and other fields, different forms of Γ have been obtained [29, 30], for example in the supersymmetry (SUSY) low temperature case, $\Gamma \simeq 0.64 \times g^2 h^4 \left(\frac{g\phi}{m_\chi}\right)^4 \frac{T^3}{(m_\chi)^3}$ ($m_\chi \approx g\phi$), which was fully calculated in Ref. [31]. Based on some different forms of dissipative coefficient, a general form of dissipative coefficient $\Gamma = C_\phi \frac{T^m}{\phi^{m-1}}$ was proposed in [32]. Here, we'll use a more general form of dissipative coefficient which is the same as in Refs. [9, 33].

$$\Gamma = \Gamma_0 \left(\frac{\phi}{\phi_0}\right)^n \left(\frac{T}{\tau_0}\right)^m, \quad (10)$$

with n and m integers and ϕ_0, τ_0, Γ_0 some nonnegative constants. This kind of general form of dissipative coefficient has the right dimension and can cover all different cases, for example, when $m = n = 0$, $\Gamma = \Gamma_0$ is recovered and when $m = 3, n = -2$, the SUSY low temperature ($\Gamma = C_\phi \frac{T^3}{\phi^2}$) case is included.

A parameter is defined to measure the damping strength of the Warm-LQC scenario

$$R = \frac{\Gamma}{3H}. \quad (11)$$

For strong dissipation regime in Warm-LQC, we have $R \gg 1$, on the contrary, $R \ll 1$ is the weak dissipation regime in Warm-LQC.

The dissipation of the inflaton's motion is associated with the production of entropy. The expression for entropy density from thermodynamics is $s = -\partial f / \partial T$, when the free energy $f = \rho - Ts$ is dominated by the potential, we have

$$s \simeq -V_T. \quad (12)$$

The total energy density, including the contributions of the inflaton and radiation, is

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi, T) + Ts. \quad (13)$$

The total pressure is given by

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi, T). \quad (14)$$

The energy-momentum conservation

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (15)$$

combining with Eq. (9) yields the entropy production equation

$$T\dot{s} + 3HTs = \Gamma\dot{\phi}^2. \quad (16)$$

If the thermal corrections to the potential is little enough, which we'll see later in the paper is the demand for a workable warm inflationary model, the radiation energy can be written as $\rho_r = 3Ts/4$, the Eq. (16) is equivalent to

$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2. \quad (17)$$

Inflation is associated with a slow-roll approximation which consists of neglecting the highest order terms in the preceding equations, which implies that the energy is potential dominated and the producing of radiation is quasi-static. The slow-roll equations in Warm-LQC are:

$$\dot{\phi} = -\frac{V_\phi}{3H(1+R)}, \quad (18)$$

$$Ts = R\dot{\phi}^2, \quad (19)$$

$$H^2 = \frac{8\pi G}{3}V(1-z), \quad (20)$$

$$4H\rho_r = \Gamma\dot{\phi}^2, \quad (21)$$

where $z = \rho/\rho_c \simeq V/\rho_c$ in Eq. (20) is a character parameter describing the quantum effect in LQC. The validity of the slow-roll approximation is dependent on a set of slow-roll parameters:

$$\epsilon = \frac{M_p^2}{2} \left(\frac{V_\phi}{V}\right)^2, \quad \eta = M_p^2 \frac{V_{\phi\phi}}{V}, \quad \beta = M_p^2 \frac{V_\phi \Gamma_\phi}{V\Gamma}, \quad (22)$$

where $M_p^2 = 1/8\pi G$. There will be two additional parameters describing the temperature dependence,

$$b = \frac{TV_{\phi T}}{V_\phi}, \quad c = \frac{T\Gamma_T}{\Gamma}. \quad (23)$$

In warm inflation, the radiation energy density and the universal temperature has the relationship:

$$\rho_r = C_r T^4. \quad (24)$$

Considering Eqs. (10), (18), (21), and (24), we can get the temperature of the universe during inflation in strong dissipation regime

$$T = \left(\frac{V_\phi^2 \phi_0^n \tau_0^m}{4HC_r \Gamma_0 \phi^n} \right)^{\frac{1}{4+m}}. \quad (25)$$

Either in low or high temperature regime, the criterion for warm inflation $T > H$ should be satisfied. When the slow roll parameter $\epsilon \simeq (1+R)(1-z)^2/(1-2z)$, $\ddot{a} = 0$, which implies the end of the inflation phase. The number of e-folds in Warm-LQC is given by

$$N(\phi) = \int H dt = -\frac{1}{M_p^2} \int_{\phi}^{\phi_e} \frac{V}{V_{\phi}} (1-z)(1+R) d\phi', \quad (26)$$

the subscript e is used to denote the end of inflation.

IV. STABILITY ANALYSIS

To find the conditions for the validity of the slow-roll approximation in the Warm-LQC scenario, we perform a linear stability analysis to see whether the system remains close to the slow-roll solution for many Hubble times. The stability is done around the slow-roll solutions, for we should obtain the conditions to guarantee they can really act as formal attractor solutions for the dynamical system. Define a new variable $u = \dot{\phi}$, so that $\dot{\phi} = \dot{u}$. We rewrite Eqs. (9) and (16) as:

$$\dot{\phi} = u, \quad (27)$$

$$\dot{u} = -3H(1+R)u - V_{\phi}, \quad (28)$$

$$T\dot{s} = -3HTs + \Gamma u^2. \quad (29)$$

Using the effective Friedmann equation, we obtain the rate of change of the Hubble parameter is

$$\frac{1}{H} \frac{d \ln H}{dt} = \frac{\dot{H}}{H^2} = -\frac{3}{2} \frac{(1-2z)(u^2 + Ts)}{(1-z)(\frac{1}{2}u^2 + Ts + V)}. \quad (30)$$

Hubble parameter is nearly a constant during slow-roll approximation period, so we require $|\dot{H}/H^2| \ll 1$. Therefore, we have conditions $Ts \ll V$ and $u^2 \ll V$, which are consistent with the inflation realizing requirement that the energy must be potential dominated. Furthermore, we should restrict the factor $\frac{1-2z}{1-z}$ is of order of unity, i.e., the quantum effect shouldn't be too large. When $\rho > \rho_c/2$, the universe is in the super-inflation phase, the character of this phase is that $\dot{H} > 0$ and changes fast, but scale factor a varies slowly. The phase is sometimes called "fast-roll" phase and the number of e-folds during that period is small [34], so we only focus on the normal inflation period.

Now we define $\delta\phi$ as a small homogeneous perturbation of the inflaton field in the slow-roll approximation and the inflaton field can be expanded as $\phi = \phi_0 + \delta\phi$, where ϕ_0 is the background part of ϕ . Similarly, we have $u = u_0 + \delta u$ and $s = s_0 + \delta s$, where the subscript '0' denotes the background part of u and s . That the perturbation parts are much less than the background terms is assumed. The equations for ϕ_0 , u_0 , and s_0 are given by

$$u_0 = -\frac{V_{\phi_0}}{3H_0(1+R_0)}, \quad (31)$$

$$T_0 s_0 = R_0 u_0^2, \quad (32)$$

$$H_0^2 = \frac{1}{3M_p^2} V_0 \left(1 - \frac{V_0}{\rho_c}\right). \quad (33)$$

Using $s \simeq -V_T$, we have

$$\delta s = -V_{TT}\delta T - V_{\phi T}\delta\phi. \quad (34)$$

As the condition that the thermal corrections to the potential is negligible, the relationship $Ts_T = 3s$ holds, with this condition, we find

$$\delta T = \frac{1}{3s_0} (T_0 \delta s + V_{\phi} b \delta\phi). \quad (35)$$

Similarly, we can calculate the variations of H , Γ , V_{ϕ} , and R in terms of the fundamental perturbation variables. The results are as follows:

$$2H_0 \delta H = \frac{1}{3M_p^2} (1-2z) (u_0 \delta u + V_{\phi} \delta\phi + T_0 \delta s), \quad (36)$$

$$\frac{\delta \Gamma}{\Gamma_0} = \left(\frac{1}{M_p^2} \frac{V_0}{V_{\phi}} \beta + \frac{c}{3s_0 T_0} V_{\phi} b \right) \delta\phi + \frac{c}{3s_0} \delta s, \quad (37)$$

$$\frac{\delta V_{\phi}}{V_{\phi}} = \left(\frac{1}{M_p^2} \frac{V_0}{V_{\phi}} \eta + \frac{V_{\phi}}{3s_0 T_0} b^2 \right) \delta\phi + \frac{b}{3s_0} \delta s, \quad (38)$$

$$\begin{aligned} \frac{\delta R}{R_0} = & \left(\frac{1}{M_p^2} \frac{V_0}{V_{\phi}} \beta + \frac{bcV_{\phi}}{3s_0 T_0} - \frac{1-2z}{2H_0^2} \frac{V_{\phi}}{3M_p^2} \right) \delta\phi \\ & - \frac{1-2z}{2H_0^2} \frac{u_0 \delta u}{3M_p^2} + \left(\frac{c}{3s_0} - \frac{1-2z}{2H_0^2} \frac{T_0}{3M_p^2} \right) \delta s \end{aligned} \quad (39)$$

The equations above will be used repeatedly thereafter.

For conciseness, we write the equations of small perturbations in a matrix form

$$\begin{pmatrix} \delta\dot{\phi} \\ \delta\dot{u} \\ \delta\dot{s} \end{pmatrix} = E \begin{pmatrix} \delta\phi \\ \delta u \\ \delta s \end{pmatrix} - F, \quad (40)$$

where E is a 3×3 matrix which can be written:

$$E = \begin{pmatrix} 0 & 1 & 0 \\ A & \lambda_1 & B \\ C & D & \lambda_2 \end{pmatrix}. \quad (41)$$

Using Eqs. (27) - (29) and Eqs. (36) - (39), we get the elements of the above matrix:

$$\begin{aligned} A = & 3H_0^2 \left[-\frac{(1+R_0)^2}{R_0} b^2 + c(1+R_0)b - \frac{\eta}{1-z} \right. \\ & \left. + \frac{\epsilon}{1+R_0} \frac{1-2z}{(1-z)^2} + \frac{R_0}{1+R_0} \frac{\beta}{1-z} \right], \end{aligned} \quad (42)$$

$$B = \frac{H_0 T_0}{u_0} \left[-c + \frac{1+R_0}{R_0} b - \frac{\epsilon}{(1+R_0)^2} \frac{1-2z}{(1-z)^2} \right], \quad (43)$$

$$C = \frac{3H_0^2 u_0}{T_0} \frac{R_0}{1+R_0} \left[\frac{(1+R_0)^2 (1-c)}{R_0} b - \beta \frac{1}{1-z} + \epsilon \frac{1-2z}{(1-z)^2} \right], \quad (44)$$

$$D = \frac{H_0 u_0}{T_0} \left[6R_0 - \frac{R_0 \epsilon}{(1+R_0)^2} \frac{1-2z}{(1-z)^2} \right], \quad (45)$$

$$\lambda_1 = -3H_0(1+R_0) - H_0 \frac{\epsilon}{(1+R_0)^2} \frac{1-2z}{(1-z)^2}, \quad (46)$$

$$\lambda_2 = -H_0(4-c) - H_0 \frac{R_0 \epsilon}{(1+R_0)^2} \frac{1-2z}{(1-z)^2}. \quad (47)$$

The "forcing term" F is a column matrix which can be expressed as

$$F = \begin{pmatrix} 0 \\ \dot{u}_0 \\ \dot{s}_0 \end{pmatrix}. \quad (48)$$

In order to make the slow-roll solution an attractor for Warm-LQC, we need the matrix E to have negative eigenvalues and the forcing term to be small enough. First we'll study the forcing term. Taking time derivatives of the Eq. (31) and (32), and after some calculation we can get

$$\dot{u}_0 = \frac{BC - A\lambda_2}{\lambda_1 \lambda_2 - BD} u_0, \quad (49)$$

$$\dot{s}_0 = \frac{AD - C\lambda_1}{\lambda_1 \lambda_2 - BD} u_0. \quad (50)$$

Combining the two equations above with expressions for the matrix elements we can finally get

$$\frac{\dot{u}_0}{H_0 u_0} = \frac{1}{\Delta} \left[-\frac{cR_0 + c - 4}{1+R_0} \epsilon \frac{1-2z}{(1-z)^2} + \frac{\eta(c-4)}{1-z} + \frac{4R_0}{1+R_0} \frac{\beta}{1-z} + 3(1+R_0)bc \right], \quad (51)$$

$$\frac{\dot{s}_0}{H_0 s_0} = \frac{3}{\Delta} \left\{ \frac{3+R_0}{1+R_0} \epsilon \frac{1-2z}{(1-z)^2} - \frac{2\eta}{1-z} + \frac{R_0-1}{R_0+1} \frac{\beta}{1-z} + [(1+R_0)^2 + c(R_0^2 - 1)] \frac{b}{R_0} \right\}. \quad (52)$$

Using Eq. (35) and the equations above, we can get

$$\frac{\dot{T}_0}{HT_0} = \frac{1}{\Delta} \left[\frac{3+R_0}{1+R_0} \epsilon \frac{1-2z}{(1-z)^2} - \frac{2\eta}{1-z} + \frac{R_0-1}{R_0+1} \frac{\beta}{1-z} - \frac{3(1+R_0)^2}{R_0} b \right], \quad (53)$$

where

$$\Delta = (4+c)R_0 + 4 - c - 2(1+R_0)b. \quad (54)$$

Furthermore, we can get the relationship for \dot{H}_0/H_0^2

$$\frac{\dot{H}_0}{H_0^2} = -\frac{1-2z}{(1-z)^2} \frac{\epsilon}{1+R_0} \ll 1. \quad (55)$$

A small forcing term requires $|\dot{u}_0/Hu_0| \ll 1$ and $|\dot{s}_0/Hs_0| \ll 1$, and through Eqs. (51)-(52), and Eqs.(54)-(55) we find sufficient conditions for these are

$$\epsilon \ll (1+R) \frac{(1-z)^2}{1-2z}, \quad |\eta| \ll (1+R)(1-z),$$

$$|\beta| \ll (1+R)(1-z), \quad |b| \ll \frac{R}{1+R}.$$

For the sake of comparison with classical warm inflation and simplicity in later sections, we redefine the slow-roll parameters in Warm-LQC

$$\tilde{\epsilon} = \epsilon \frac{1-2z}{(1-z)^2}, \quad \tilde{\eta} = \eta \frac{1}{1-z}, \quad \tilde{\beta} = \beta \frac{1}{1-z}. \quad (56)$$

Using the new parameters, the slow-roll conditions in Warm-LQC will have the same form with the classical case ($\tilde{\epsilon} \ll 1+R$, $\tilde{\eta} \ll 1+R$ and $\tilde{\beta} \ll 1+R$) [35]. Consistently, in the limit $\rho_c \rightarrow \infty$, naturally we have $\tilde{\epsilon} \rightarrow \epsilon$, $\tilde{\eta} \rightarrow \eta$, $\tilde{\beta} \rightarrow \beta$, i.e. the new defined slow-roll parameters are reduced to the classical ones.

During normal inflation phase in LQC ($0 < z < 1/2$), $(1-z)^2/(1-2z) > 1$, so the condition for ϵ is less restricted and inflation can last longer than in the classical case, but the conditions for η and β are more restricted due to the quantum correction. Furthermore, the range of slow-roll parameters ϵ and η are all enlarged by a factor $(1+R)$ compared to standard inflation (the slow-roll condition in standard inflations are $\epsilon \ll 1$ and $\eta \ll 1$ [3, 36]). The slow-roll parameter b is much more small than others, which indicates a negligible thermal correction to the potential. The condition for b guarantees the equivalency between Eqs. (16) and (17). With a small b , we have

$$\Delta \approx (4+c)R_0 + 4 - c. \quad (57)$$

Now we will find conditions for the matrix having negative or positive but of order $\mathcal{O}(\epsilon/R)$ eigenvalues. Through the slow-roll conditions we have obtained, we find the matrix elements A and C are very small, so we have the

characteristic equation for the matrix E as

$$\begin{aligned} \det(\lambda I - E) &\simeq \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & \lambda_1 - \lambda & B \\ 0 & D & \lambda_2 - \lambda \end{vmatrix} \\ &= -\lambda(\lambda_1 - \lambda)(\lambda_2 - \lambda) + BD\lambda \\ &= 0. \end{aligned} \quad (58)$$

Obviously there is a very small eigenvalue $\lambda \ll \lambda_1, \lambda_2$:

$$\lambda \simeq \frac{-BC - A\lambda_2}{\lambda_1\lambda_2 - BD - A}. \quad (59)$$

The other two eigenvalues satisfy

$$\lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2 - BD = 0. \quad (60)$$

Both eigenvalues are negative if $\lambda_1 + \lambda_2 < 0$, $\lambda_1\lambda_2 - BD > 0$ are satisfied. Substituting the expression for λ_1 , λ_2 , B and D we can finally get

$$|c| < 4. \quad (61)$$

Now we have obtained all slow-roll conditions in the Warm-LQC scenario. We find that the conditions for ϵ , η , and β are modified by the quantum correction compared to warm inflation in classical universe [35], and the conditions for b and c are the same as in [35] for they only represent the temperature dependence in inflation.

In the next two sections, we will discuss in the regime where the slow-roll conditions are satisfied. The slow-roll condition for ϵ naturally guarantees the potential dominating by

$$\frac{\rho_r}{V} = \frac{R\epsilon}{2(1+R)^2(1-z)} \ll 1. \quad (62)$$

V. THE CASE OF $\Gamma = \Gamma(\phi)$

In this section, we will focus on the case where the dissipative coefficient is independent of temperature, i.e.

$$\Gamma = \Gamma(\phi). \quad (63)$$

This is the case for $m = 0$ in the Eq. (10) and using the definition of c , we can see the slow-roll parameter $c = m = 0$ here and the expression for temperature is reduced to

$$T = \left(\frac{V_\phi^2 \phi_0^n}{4HC_r \Gamma_0 \phi^n} \right)^{1/4}. \quad (64)$$

In the $m = 0$ case, the spectrum of the field perturbation amplitude caused by thermal fluctuations can be written in an analytical form [11]

$$P_\phi = \frac{\sqrt{3\pi}}{2} HT \sqrt{1+R}. \quad (65)$$

The complicated treatment of perturbations of the effective Hamiltonian in LQC in details can be seen in [25, 37], which is beyond the scope of our paper. And we point out here that the cosmological perturbations in LQC have been investigated systematically in many references such as [20, 25, 37]. As stated in [20, 22], the quantum effect is weak when horizon crossing, which we will see in our later paragraph also. So for simplicity, we will follow the treatment in Ref. [20, 22] for LQC, the corresponding curvature perturbation is given by $\mathcal{R} = \left(\frac{H}{\dot{\phi}} \right) \delta\phi$, so we can get the amplitude of scalar perturbation as

$$P_s = \left(\frac{H}{\dot{\phi}} \right)^2 P_\phi = \frac{\sqrt{3\pi} H^3 T}{2 \dot{\phi}^2} \sqrt{1+R}, \quad (66)$$

which is valued at the Hubble radius crossing $k = aH$. We can see that $\dot{\phi}$ is in the denominator, and we may ask what will happen if $\dot{\phi} = 0$. $\dot{\phi}$ may vanish when the loop quantum effect driving the inflaton to the potential hill during the non-slow-roll regime in loop quantum scenario. Fortunately, as stated in [38], the expression for scalar perturbation behaves well even when $\dot{\phi} = 0$, and the expression of P_s

$$P_s = \frac{\sqrt{3\pi} 9H^5 T}{2 V_\phi^2} (1+R)^{\frac{5}{2}} \quad (67)$$

is still valid. Furthermore, the validity of slow-roll conditions keep us away from the $\dot{\phi} = 0$ case. Substituting Eqs. (18), (20) and (64) to the equation above yields:

$$\begin{aligned} P_s &= \sqrt{\frac{243\pi}{8}} (1+R)^{2/5} V_\phi^{-3/2} \\ &\times \left[\Gamma_0 C_r \left(\frac{\phi}{\phi_0} \right)^n \right]^{-1/4} \left[\frac{V(1-z)}{3M_p^2} \right]^{19/8}. \end{aligned} \quad (68)$$

The spectral index is defined by

$$n_s - 1 = \frac{d \ln P_s}{d \ln k}. \quad (69)$$

Since H varies slowly during the inflation, we have $d \ln k = d(\ln aH) \simeq d \ln a = H dt$, and then

$$n_s - 1 = \frac{\dot{P}_s}{HP_s}. \quad (70)$$

By virtue of Eq. (39) and Eqs. (51)-(55), we obtain

$$\begin{aligned} &n_s - 1 \\ &= \frac{1}{\Delta} \left[-\frac{9R - 5c + 17}{1+Q} \epsilon \frac{1-2z}{(1-z)^2} - \frac{9R+1}{1+R} \frac{\beta}{1-z} \right. \\ &\quad + \frac{-3cR + 6R + 6 - 2c}{1+R} \frac{\eta}{1-z} \\ &\quad \left. - \frac{3(1+R)(5cR + 2R + 2)}{2R} b \right]. \end{aligned} \quad (71)$$

In the case $c = 0$, by using new slow-roll parameter defined in Eq. (56) the equation above can reduce to

$$n_s - 1 = \frac{1}{\Delta} \left[-\frac{9R+17}{1+R}\tilde{\epsilon} + 6\tilde{\eta} - \frac{9R+1}{1+R}\tilde{\beta} - \frac{3(1+R)^2}{R}b \right]. \quad (72)$$

From the slow-roll conditions we have obtained, we find $n_s \approx 1$, i.e. we have a nearly scale-invariant power spectrum which is consistent with observation.

As stated in Sec. III, we have $R \gg 1$ in the strong regime of Warm-LQC, thus

$$n_s - 1 = -\frac{9}{4R}\tilde{\epsilon} - \frac{9}{4R}\tilde{\beta} + \frac{3}{2R}\tilde{\eta} - \frac{3}{4}b, \quad (73)$$

which agrees with a partial result in Ref. [9] in form. In the weak regime of Warm-LQC ($R \ll 1$), thermal fluctuations lead to the spectral index

$$n_s - 1 = -\frac{17}{4}\tilde{\epsilon} + \frac{3}{2}\tilde{\eta} - \frac{1}{4}\tilde{\beta} - \frac{3}{R}b. \quad (74)$$

Although in the weak regime, the spectral index does not approach that in the standard inflation when $R \rightarrow 0$, the reason we will see in later section.

Now we will calculate the running of the spectral $\alpha_s = \frac{dn_s}{d \ln k} = \frac{dn_s}{H dt}$. For simplicity, we only focus on the strong regime, where the running of the spectral can be expressed as

$$\alpha_s = \frac{1}{R} \left[-\frac{9}{4}\tilde{\epsilon}' - \frac{9}{4}\tilde{\beta}' + \frac{3}{2}\tilde{\eta}' \right] - \frac{R'}{R} \left(n_s - 1 + \frac{3}{4}b \right) - \frac{3}{4}b', \quad (75)$$

where

$$\tilde{\epsilon}' = \frac{8z^2\epsilon^2(2\epsilon - \eta)}{(1-z)^5(1+R)^2}, \quad (76)$$

$$\tilde{\eta}' = \frac{2\epsilon^2z(\xi - 2\eta)}{(1-z)^4(1+R)^2}, \quad (77)$$

$$\tilde{\beta}' = \frac{2\epsilon z(2\epsilon\sigma + \beta\eta - 2\epsilon\beta - 2\epsilon\delta)}{(1-z)^4(1+R)^2}, \quad (78)$$

$$R' = -\frac{R\beta}{(1+R)(1-z)} + \frac{R(1-2z)\epsilon}{(1+R)(1-z)^2}, \quad (79)$$

$$b' = \frac{b\eta}{(1-z)(1+R)} - \frac{\zeta}{(1-z)(1+R)}. \quad (80)$$

In the equations above, we have $' = d/d \ln k$, $\xi = 2M_p^2 V_{\phi\phi\phi}/V$, $\sigma = M_p^2 \Gamma_{\phi\phi}/\Gamma$, $\delta = M_p^2 \Gamma_{\phi}^2/\Gamma^2$, $\zeta =$

$M_p^2 TV_{\phi\phi T}/V$, and $\xi, \sigma, \delta, \zeta$ are all small quantities. If we use λ to refer to slow-roll parameters in general in this section, we find in strong regime of Warm-LQC, the spectral index is of order $\mathcal{O}(\lambda/R)$, and the running of the spectral is of order $\mathcal{O}(\lambda^2/R^2)$ (the dominated term is the second one in the expression of α_s and α_s is a little smaller by the quantum correction). As $R \gg 1$, we probably get a nearly scale-invariant spectrum.

Tensor perturbations do not couple strongly to the thermal background, and so gravitational waves are only generated by the quantum fluctuations, as in standard inflation [28],

$$P_T = \frac{2}{M_p^2} \left(\frac{H}{2\pi} \right)^2 = \frac{V(1-z)}{6\pi^2 M_p^4} \sim \frac{V}{M_p^4}. \quad (81)$$

Amusingly, we can see from the equation above that the gravitational wave spectrum and the quantum effect ($z \simeq V/\rho_c$) have the same order. If the primordial tensor perturbations are seen in the CMB sky, so does the quantum effect.

The spectral index of tensor perturbation is

$$n_T = \frac{\dot{P}_T}{HP_T} = -\frac{2\tilde{\epsilon}}{1+R}, \quad (82)$$

and the tensor-to-scalar ratio is

$$r = \frac{P_T}{P_s} = \frac{2H\epsilon}{\sqrt{3}\pi^{5/2}(1+R)^{5/2}T(1-z)^2}, \quad (83)$$

which is much smaller than in standard inflation due to the factor $(1+R)^{5/2}$ in the denominator.

Through the analysis above, we can find that both scalar and tensor perturbations are depressed by loop quantum correction to varying degrees compared to the classical case. We can get a nearly scale-invariant power spectrum for warm inflation in both classical and loop cosmology. The expression for spectral index n_s and n_T are modified by quantum correction but has the similar form with those in classical universe [35]. But the expression for the running of spectral α_s is very different from the classical one [32], the loop correction can result in some additional terms but doesn't change the magnitude of the running of spectral. Furthermore, the tensor-to-scalar ratio is a little larger by a factor $1/(1-z)^{3/2}$ than in classical warm inflation.

Now we will study the amplitude of quantum effect and see whether it can be observed. The expected amplitude of the quantum effect can be characterized by a parameter z which appears in the power spectrum of Warm-LQC, and we can determine its value by fitting the theoretical results to observational data. The WMAP observation results give the values for scalar power spectrum $P_s \simeq 2.3 \times 10^{-9}$ and the tensor-to-scalar ratio $r < 0.2$, and by using Eq. (81) we can obtain $z = \frac{V}{\rho_c} < \frac{0.2}{0.41} \frac{6\pi^2}{1-z} P_s$ is a small quantity when horizon crossing, so we can use the condition to simplify the calculation in the following.

Using Eq. (67) we get

$$z = \frac{P_s^2 \tilde{\epsilon}^2}{4 \times 0.41 \pi^3 (1+R)^5} \left(\frac{M_p}{T} \right)^2$$

$$\simeq 10^{-18} \frac{\tilde{\epsilon}^2}{(1+R)^5} \left(\frac{M_p}{T} \right)^2, \quad (84)$$

which is fairly depressed by the factor 10^{-18} and $(1+R)^5$ in the denominator. As in standard inflation in LQC [20] and warm inflation in LQC [22], we also come to the conclusion that the LQC quantum effect is too tiny to be observed today. For $R \gg 1$, we have

$$z \ll 10^{-18} \tilde{\epsilon}^2 \left(\frac{M_p}{T} \right)^2. \quad (85)$$

The upper limit for z is dependent on temperature and $\tilde{\epsilon}$ and the upper limit increases when temperature decreases.

A separable potential example

Let us consider a separable potential in SUSY case [33]:

$$V(\phi, T) = v_1(T) + v_2(\phi), \quad (86)$$

where

$$v_1(T) = -\frac{\pi^2}{90} g_* T^4 - \frac{1}{12} m_\phi^2 T^2, \quad (87)$$

$$v_2(\phi) = \frac{1}{2} m_\phi^2 \phi^2. \quad (88)$$

SUSY is a good mechanism which can suppress the thermal corrections to the potential, so in this case, the potential can be written in a separable form. The slow-roll parameter $b = 0$ here can certainly satisfy the slow-roll conditions and the other slow-roll parameters are given by

$$\epsilon = \frac{M_p^2 m_\phi^4 \phi^4}{2V^2}, \quad \eta = \frac{M_p^2 m_\phi^2}{V}, \quad \beta = \frac{n M_p^2 m_\phi^2}{V}. \quad (89)$$

Because of Eq. (62), we have $V \simeq v_2$, so the three parameters above can be written as

$$\epsilon \simeq \frac{2M_p^2}{\phi^2}, \quad \eta \simeq \frac{2M_p^2}{\phi^2}, \quad \beta \simeq \frac{2nM_p^2}{\phi^2}. \quad (90)$$

With this potential, the slow-roll conditions hold in the regime $\phi \gg \frac{M_p}{\sqrt{1+R}}$. If the R is big enough, the lower bound can be much smaller than Planck scale, while in the standard inflation case with a chaotic potential, the range of the inflaton is $\sqrt{2}M_p < \phi < \sqrt{4N+2}M_p$ (N is the number of e-folds), which is too big to build a consistent inflationary model [8, 36]. Many kinds of monomial potential in standard inflation suffer the problem

of having an overlarge inflaton amplitude, which can be eliminated in a warm inflation scenario. Hence the warm inflation can take place for a wider range of potentials than standard inflation as the slow-roll conditions imply.

Using $\rho_r = 3Ts/4$ and the expression for the separable potential, we have $C_r = \frac{\pi^2}{30} g_*$ (g_* is a constant and $g_* \approx 100$ for the radiation field) in Eq. (24) and the strength of dissipation parameter is given by

$$R = \frac{M_p \Gamma_0 (\phi/\phi_0)^n}{\sqrt{\frac{3}{2}} (1-z) m_\phi \phi}. \quad (91)$$

By using $V_\phi = m_\phi^2 \phi$ and Eq. (64) we obtain the scalar power spectrum in a concrete form

$$P_s = \sqrt{\frac{243\pi}{8}} \left(\frac{1}{3} \right)^{19/8} (1+R)^{5/2} (1-z)^{19/8}$$

$$\times C_r^{-1/4} \left(\frac{\phi_0}{\phi} \right)^{n/4} \left(\frac{M_p}{m_\phi} \right)^3 \left(\frac{M_p}{\phi} \right)^{3/2}$$

$$\times \left(\frac{M_p}{\Gamma_0} \right)^{1/4} \left(\frac{V}{M_p^4} \right)^{19/8}. \quad (92)$$

The last factor must be very small to obtain an observation consistent P_s with many factors ahead are big, so the quantum effect characteristic parameter is a tiny one. By using Eq. (26) we can estimate the amplitude of inflaton when horizon crossing $k = aH$ (denoted by ϕ_*), $\phi_* \sim \sqrt{(4N+2)/(1+R)} M_p$. In strong dissipation regime, we take $R \simeq 10^2$, $g_* \sim 100$, $\Gamma_0 \simeq 10^{-6} M_p$, $\phi_0 \sim M_p$ [33], then we have $\phi_* \sim M_p$. Setting P_s to the observed value $\approx 2 \times 10^{-9}$ and using the parameters fixed above leads to $m_\phi \simeq 10^{-8} M_p$.

The spectral index is given by

$$n_s - 1 = \frac{1}{1+R} \frac{M_p^2}{2\phi^2} \left[-\frac{9R+17}{1+R} \frac{1-2z}{(1-z)^2} \right. \\ \left. + \frac{6}{1-z} - \frac{9R+1}{1+R} \frac{n}{1-z} \right], \quad (93)$$

which is of order $\mathcal{O}(M_p^2/R\phi^2)$, and a nearly scale-invariant power spectrum is guaranteed by slow-roll conditions.

In the strong dissipation regime:

$$n_s - 1 = \frac{-3}{2R(1-z)} \frac{M_p^2}{\phi^2} (3n+1). \quad (94)$$

When $n > -1/3$, the spectrum is red, and when $n < -1/3$, the spectrum can be blue. The power spectrum of tensor perturbation is

$$P_T = \frac{V(\phi, T)(1-z)}{6\pi^2 M_p^4} \simeq \frac{m_\phi^2 \phi_*^2 (1-z)}{12\pi^2 M_p^4}. \quad (95)$$

We can see from the equation above that temperature may leave its imprint on tensor perturbation in Warm-LQC scenario, while in standard inflation tensor perturbation is zero-temperature. This effect should be very tiny since tensor perturbation itself is small.

The number of e-folds is given by

$$N = -\frac{1}{M_p^2} \int_{\phi_*}^{\phi_e} \frac{\phi}{2} \left(1 - \frac{m_\phi^2 \phi^2}{2\rho_c} \right) (1+R) d\phi' \approx \frac{(1+R)\phi_*^2}{4M_p^2}, \quad (96)$$

where we use the conditions that z is small after horizon crossing and $\phi_* \gg \phi_e$ (ϕ_e is the amplitude of inflaton at the end of inflation $\phi_e \approx M_p/\sqrt{1+R}$).

VI. THE CASE OF $\Gamma = \Gamma(\phi, T)$

In this section, we will focus on the case where the dissipative coefficient is dependent of temperature, i.e.

$$\Gamma = \Gamma(\phi, T). \quad (97)$$

This is the case for $m \neq 0$ in the Eq. (10) and by definition $c = m$. The temperature is given by Eq. (25).

In this case the fluctuation power spectrum P_ϕ is determined by two coupled equations, and it's hardly to get analytic results [9]. Previously most work about warm inflation focuses on the power spectrum of the temperature independent case shown by Eq. (66). In Ref. [11], the authors pay attention to the temperature dependent case and give the analytic approximation solution and numerical simulation. Here we take the new form of approximation solution for the power spectrum proposed in Ref. [11] which should be used in the $\Gamma_T \neq 0$ case:

$$P_s = \left(\frac{H}{\dot{\phi}} \right)^2 P_\phi = \frac{\sqrt{3}\pi}{2} \frac{H^3 T}{\dot{\phi}^2} \sqrt{1+R} \left(1 + \frac{R}{r_c} \right)^{3c}, \quad (98)$$

where r_c varies slowly in the range $0 \leq c \leq 3$, for example $r_1 \approx 8.53$, $r_2 \approx 7.66$, $r_3 \approx 7.27$. The formula is consistent with the old one when $c = m = 0$. The $\dot{\phi} = 0$ case doesn't affect our analysis as stated in the previous chapter. After some cockamamie calculation we can obtain the spectral index for the new scalar power spectrum:

$$\begin{aligned} n_s - 1 &= \frac{1}{\Delta} \left\{ \left[\frac{-17+5m-9R}{1+R} + \frac{6mR(m+2)}{1+R/r_c} \right] \tilde{\epsilon} \right. \\ &+ \left(\frac{-3mR+6R-2m+6}{1+R} - \frac{6m^2R}{1+R/r_c} \right) \tilde{\eta} \\ &- \left(\frac{9R+1}{1+R} + \frac{12mR}{1+R/r_c} \right) \tilde{\beta} \\ &\left. - \left[\frac{3(5mR+2R+2)}{2R} + \frac{9m^2(1+R)}{1+R/r_c} \right] (1+R)b \right\}. \end{aligned} \quad (99)$$

Many new terms are added compared to Eq. (72) but we still have $n_s \approx 1$ as the slow-roll conditions hold. We can

express the additional terms as

$$\begin{aligned} \Delta n_s &= \frac{1}{\Delta(1+R/r_c)} [6mR(m+2)\tilde{\epsilon} - 6m^2R\tilde{\eta} \\ &- 12mR\tilde{\beta} - 9m^2(1+R)^2b] + \frac{1}{\Delta} \left[\frac{5m}{1+R}\tilde{\epsilon} \right. \\ &\left. - \frac{3mR+2m}{1+R}\tilde{\eta} - \frac{15m(1+R)}{2} \right], \end{aligned} \quad (100)$$

when $m \rightarrow 0$, $\Delta n_s \rightarrow 0$.

In the strong dissipation regime:

$$\begin{aligned} n_s - 1 &= \frac{1}{(4+m)R} \{ [-9+6r_cm(m+2)]\tilde{\epsilon} \\ &+ (-3m+6-6r_cm^2)\tilde{\eta} - (9+12r_cm)\tilde{\beta} \\ &- \left[\frac{3}{2}(5mR+2R) - 9r_cm^2R \right] b \}, \end{aligned} \quad (101)$$

$$\begin{aligned} \Delta n_s &= \frac{m}{(4+m)R} [6r_c(m+2)\tilde{\epsilon} - (6r_cm-3)\tilde{\eta} \\ &- 12r_c\tilde{\beta} - \left(9r_cmR + \frac{15}{2}R \right) b], \end{aligned} \quad (102)$$

where we can see that Δn_s is of order $\mathcal{O}(\lambda r_c/R)$. With the new form of the power spectrum we still have a nearly scale-invariant spectrum but the departure from 1 for spectral index is slightly bigger than the old one. In this section, we refer "old" as the results obtained by using the power spectrum given by Eq. (66).

In the weak dissipation regime:

$$\begin{aligned} n_s - 1 &= \frac{1}{4-m} [(-17+5m)\tilde{\epsilon} - \tilde{\beta} \\ &+ (6-2m)\tilde{\eta} - \left(\frac{3}{R} + 9m^2 \right) b], \end{aligned} \quad (103)$$

$$\Delta n_s = \frac{m}{4-m} \left(5\tilde{\epsilon} - 2\tilde{\eta} - \frac{15}{2}b - 9bm \right). \quad (104)$$

The running of the spectral in strong dissipation regime is given by

$$\begin{aligned} \alpha_s &= \frac{1}{(4+m)R} \{ [(-9+6r_cm(m+2))\tilde{\epsilon}' \\ &+ (-3m+6-6r_cm^2)\tilde{\eta}' - (9+12r_cm)\tilde{\beta}' \\ &- \left(\frac{3}{2}(5m+2) + 9r_cm^2 \right) R] b' \\ &- \left[\frac{3}{2}(5m+2) + 9r_cm^2 \right] bR' \} \\ &- (n_s - 1) \frac{R'}{R}, \end{aligned} \quad (105)$$

where the parameters are already given in the preceding part of the paper. The running is still of order $\mathcal{O}(\lambda^2/R^2)$.

Tensor perturbation is not coupled to the radiation, so the tensor power spectrum does not change. The tensor-to-scalar ratio is given by

$$r = \frac{P_T}{P_s} = \frac{2H\epsilon(1+R/r_c)^{-3m}}{\sqrt{3}\pi^{5/2}(1+R)^{5/2}T(1-z)^2}, \quad (106)$$

which is smaller than the old one if $m > 0$.

There are also some differences between warm inflation in classical and loop cosmology, and most are the same as we stated in the Sec. V. So we don't state again here to avoid repetition.

The quantum characteristic parameter z has the amplitude constrained by WMAP observations

$$\begin{aligned} z &\simeq 10^{-18} \frac{\tilde{\epsilon}^2}{(1+R)^5} \left(\frac{M_p}{T}\right)^2 \left(1 + \frac{R}{r_c}\right)^{-6m} \\ &\ll 10^{-18} \tilde{\epsilon}^2 \left(\frac{M_p}{T}\right)^2 \left(1 + \frac{R}{r_c}\right)^{-6m}, \end{aligned} \quad (107)$$

which is depressed by a factor $(1+R/r_c)^{-6m}$ if $m > 0$, so the energy scale is lower than the old case.

From the above analysis, we reach the conclusion that for $m > 0$, the new form of the power spectrum gives a slightly bigger spectral index, a lower tensor-to-scalar ratio and the lower energy scale when horizon crossing.

A separable potential example

In order to compare to the $m = 0$ case, here we also take the potential in Eq. (86) in SUSY background. The slow-roll parameters are all the same as in the old case, we don't list them here to avoid repetition and the expression for the strength of dissipation parameter is

$$R = \frac{M_p \Gamma_0 (\phi/\phi_0)^n (T/\tau_0)^m}{\sqrt{\frac{3}{2}(1-z)m_\phi \phi}}. \quad (108)$$

The scalar power spectrum in a concrete form is given by

$$\begin{aligned} P_s &= \frac{9\sqrt{3}\pi}{2} \left(\frac{1}{4}\right)^{\frac{1}{4+m}} \left(\frac{1}{6}\right)^{\frac{5m+19}{2(4+m)}} (1+R)^{5/2} C_r^{-1/(4+m)} \\ &\times \left(1 + \frac{R}{r_c}\right)^{3m} (1-z)^{\frac{5m+19}{2(4+m)}} \left(\frac{\phi_0}{\phi}\right)^{\frac{n}{4+m}} \left(\frac{m_\phi}{M_p}\right)^{\frac{m+7}{m+4}} \\ &\times \left(\frac{\phi}{M_p}\right)^{\frac{3m+13}{4+m}} \left(\frac{\tau_0}{M_p}\right)^{\frac{m}{4+m}} \left(\frac{M_p}{\Gamma_0}\right)^{\frac{1}{4+m}} \end{aligned} \quad (109)$$

For example, when $m = 3$, using the parameters fixed at the same value as in the previous section and setting P_s to the observed value $\approx 2 \times 10^{-9}$ finally lead to $m_\phi \simeq 10^{-14} M_p$. The constraint is stronger than $m = 0$ case ($m_\phi = 10^{-8} M_p$), and even when $m = 0$ in Warm-LQC, the constraint is stronger than that in standard inflation where $m_\phi \simeq 10^{-7} M_p$ [36]. We can conclude that for the chaotic potential case, Warm-LQC with $\Gamma \propto T^m$ ($m > 0$)

requires a smaller inflaton mass, which is consistent with our previous analysis that the new power spectrum with $m > 0$ depresses the energy scale of inflation.

The spectral index is

$$\begin{aligned} n_s - 1 &= \frac{2M_p^2}{\Delta\phi^2} \left[\left(\frac{-17-9R+5m}{1+R} + \frac{6mR(m+2)}{1+R/r_c} \right) \frac{1-2z}{(1-z)^2} \right. \\ &\quad + \left(\frac{-3mR+6R-2m+6}{1+R} - \frac{6m^2R}{1+R/r_c} \right) \frac{1}{1-z} \\ &\quad \left. - \left(\frac{9R+1}{1+R} + \frac{12mR}{1+R/r_c} \right) \frac{n}{1-z} \right], \end{aligned} \quad (110)$$

where the part in the [] are of order of unity, with slow-roll conditions hold, the factor $2M_p^2/\Delta\phi^2 \ll 1$, so we have $n_s \approx 1$.

In strong dissipation regime:

$$\begin{aligned} n_s - 1 &= \frac{6M_p^2}{(4+m)R\phi^2(1-z)} \\ &\times [4r_cm(1-n) - m - 3n - 1]. \end{aligned} \quad (111)$$

when $m = 3$, the spectrum can be red when $n > 1$ and blue when $n < 1$.

The condition for warm inflation is $T > H$. We now analyze whether the ratio T/H will be larger in $m > 0$ case. We rewrite the scalar perturbation Eq. (98) as

$$P_s \approx \frac{T^4 H^3}{u^2 T^3} (1+R)^{1/2} \left(1 + \frac{R}{r_c}\right)^{3m}. \quad (112)$$

Using Eq. (32) and the potential expression Eq. (86) we can obtain:

$$\frac{T}{H} \approx \left(\frac{45}{2\pi^2 g_*}\right)^{1/3} (1+R)^{1/6} \left(1 + \frac{R}{r_c}\right)^m \left(\frac{R}{P_s}\right)^{1/3}. \quad (113)$$

The ratio T/H is larger by a factor $\left(1 + \frac{R}{r_c}\right)^m$, when $m > 0$, so the thermal effect is more remarkable when dissipative coefficient has the form $\Gamma \propto T^m$ with $m > 0$. And $m < 0$ case will have the opposite behavior. Since $m < 0$ corresponds to a non-SUSY case, we have paid little attention to that case.

The condition for warm inflation $T > H$ can be certainly satisfied by

$$R > g_* P_s, \quad (114)$$

where g_* is of order $\mathcal{O}(10^2)$, and P_s is of order $\mathcal{O}(10^{-9})$, so very weak dissipation can result warm inflation. Although in the weak dissipation regime, Warm-LQC results are still different from standard inflation in LQC such as Eq. (74).

VII. CONCLUSION

In this paper, we have investigated the warm inflationary scenario in LQC. We use the general form of dissipa-

tive coefficient $\Gamma = \Gamma_0 (\phi/\phi_0)^n (T/\tau_0)^m$ to study Warm-LQC for the first time. We give a short review of the effective theory of LQC and the framework of Warm-LQC. In the scenario of Warm-LQC, the universe has the temperature shown by Eq. (25). We consider the most general and reasonable case that the potential and dissipative coefficient are temperature dependent, so we introduced five slow-roll parameters in Warm-LQC.

We perform a linear stability analysis to determine the conditions that are sufficient for the system to keep the slow-roll solution as an attractor. We have proved that the consistency of the slow-roll Warm-LQC requires the slow-roll parameters satisfy: $\tilde{\epsilon} \ll 1 + R$, $|\tilde{\eta}| \ll 1 + R$, $|\tilde{\beta}| \ll 1 + R$ and $|b| \ll R/(1 + R)$, $|c| < 4$. The first three parameters ($\tilde{\epsilon}$, $\tilde{\eta}$, $\tilde{\beta}$) are new defined in Warm-LQC, the slow-roll conditions given by them imply the slow-roll inflation couldn't happen in the super-inflation phase. The condition on b implies Warm-LQC is only possible when a mechanism such as SUSY, suppresses thermal corrections to the potential. The conditions on b and c are proved to be the same as in classical warm inflation for they only describe the temperature dependence.

We study two cases when the dissipative coefficient is independent ($m = 0$) and dependent ($m \neq 0$) on temperature. We use different expressions of power spectrum to investigate the two cases and obtain their spectral index and the running of spectral that are different in form. The power spectrum in the temperature independent case we used is the same as [22], which also treats the warm inflationary models in loop quantum cosmology. And we don't treat perturbations of the effective Hamiltonian in loop cosmology in detail. We give a separable potential example which satisfies the negligible thermal correction condition for both cases. By using the new power spectrum for the $m \neq 0$ case, we find that the perturbation amplitude is enhanced when $m > 0$ and the contribution of thermal fluctuations to density fluctuations is more outstanding. And the temperature

dependence for $m > 0$ depresses the energy scale of inflation when horizon crossing compared to models with $m = 0$. In both cases we have $n_s \approx 1$, but a bigger spectral index and a smaller tensor-to-scalar ratio is obtained in $m > 0$ case. We also find the ratio T/H is enhanced when $m > 0$, so the thermal effect is more obvious. The differences between warm inflation in classical and loop universality can be seen from the expression for the power spectrum and spectral index, etc. They all acquire modifications described by the quantum parameter z . Among the modifications, the most significant one is that the tensor-to-scalar ratio is larger by $1/(1 - z)^{3/2}$. And the running of the spectral has a more complicated expression and is a little smaller than the one in classical universe. We investigated the quantum effect characterized by a parameter z in both cases, although quantum effect is dominated in very early universe, but it leaves tiny imprint on the CMB sky and can be hardly observed today.

We should note that there are some other properties of this model that are not considered in this paper but deserve further study. For example, the non-Gaussian effects during warm inflation [39] is an important signature which should be analyzed in Warm-LQC scenario. Furthermore, during the warm inflation phase, the universe is a multi-component system and the nonadiabatic entropy perturbation should be present [40] and can sometimes leave behind an impression on the curvature fluctuations [35]. These should be considered in Warm-LQC scenario and can be our future work.

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